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Robust Control Invariant Set of Linear Systems with Disturbances

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***Abstract*—In this paper the schemes to compute robust control invariant sets for linear (uncertain) systems are collected, which are not necessary minimal robust control invariant sets. Firstly, the concept of logarithmic norm is resorted to a simply differential functional inequality. Feedback control laws are obtained for both of the two schemes. In the scheme based on the differential functional inequality, the obtained robust control invariant set is a level set of the storage function.**

I. Introduction

Robust control invariant set refers to a bounded state space region in which the system state can be confined, despite the presence of disturbances or uncertainties, through the application of a control law [1, 2]. Invariant set plays an important role in the robustness analysis and synthesis of controllers for uncertain systems [3–9]. Furthermore, terminal set, which is an invariant set of considered systems under a linear control law, is needed in the formulation of model predictive control (MPC) [10, 11]. Both recursive feasibility and asymptotic stability can be assured by the appropriately chosen terminal set and the related linear control law [12–14]. Tube MPC [15–19] considers minimization of a nominal cost functional rather than a worst case one, while imposing the fulfillment of constraints for all admissible disturbances. The algorithms utilize both a state feedback control law and a control action. The control action which is calculated online, steers the nominal system state to the equilibrium. The state feedback control law keeps the actual trajectory of the constrained system in robust control invariant sets centered along the nominal trajectory.

The effect of disturbances or perturbations is a common issue related to analysis and synthesis of dynamical systems [20, 21]. In a typical situation, the value of the perturbations or disturbances is unknown but bounded. If the perturbations or disturbances are non-vanishing, i.e., do not disappear while the state is close to the equilibrium, asymptotic stability is in general not possible. However, under certain conditions, robust control invariant of the systems can be guaranteed. Recently, many research efforts have been directed to the problem of computing robust invariant sets. The approximation of the minimal robust invariant set of an asymptotically stable discrete time linear time-invariant system is considered in [22], which allows one to a priori specify the accuracy of the approximation. Compared with linear control law [22], nonlinear control law (piecewise affine in the most frequently encountered cases) is adopted [23], where the existence of two families of robust control invariant sets is established. An algorithm for the computation of robust control invariant sets for linear discrete-time systems subject to norm-bounded model uncertainty, additive disturbances and polytopic constraints on the input and state is proposed in [2]. The proposed scheme explicitly takes account of norm-bounded model uncertainty and does not require any iterative computations or initial estimates of the invariant set. The construction of robust control invariant sets of systems with matched nonlinearity and a particular class of piecewise affine systems are exploited in [24]. The maximum controlled invariant set for discrete-time systems is characterized as the solution of an infinite-dimensional linear programming problem [25], and finite-dimensional approximations of the dual of which provide a converging sequence of semi-algebraic outer approximations of this set. A real function is called a D. C. function if it is a difference of two convex functions. A method for computing a convex robust control invariant set for discrete-time nonlinear uncertain systems is presented [26], exploiting the properties of D. C. functions.

In this paper, algorithms for computing robust control invariant sets are proposed. Exploiting the properties of logarithmic norm, a condition for the robust control invariant set of linear time-invariant systems is provided. It shows that if the controlled linear systems is exponentially stable, there exists a robust control invariant set. Based on a functional inequality, robust control invariant sets of linear systems with perturbations and disturbances are considered, where both polytopic and norm-bounded linear differential inclusions are used to model the uncertainty of the linear systems. The proposed method is interesting for two reasons. On one hand, it indicates further robust control invariant sets are useful tool for controller synthesis of linear uncertain systems, and it guarantees robust stability for an adequate set of initial conditions. On the other hand, it provides a fairly simple algorithmic procedure.

This paper is organized as follows. Section II concerns with the system description and the definition of robust control invariant sets and robust invariant sets. Section III deals with the problem of calculating a robust control invariant set for linear time-invariant system by logarithmic norm. Section IV reviews a general scheme to compute robust control invariant sets, and a numerical point of view on it. Section V discusses robust control invariant set of linear uncertain systems, namely polytopic and norm-bounded uncertain systems. Finally, Section VI presents conclusions.

A. Notations and Basic Definitions

Let $\mathbb{R}$ denote the field of real numbers, $\mathbb{R}^n$ the n-dimensional Euclidean space, $\mathbb{Z}$ denote the set of integer numbers. For a vector $v \in \mathbb{R}^n$, $\|v\|$ denotes the 2-norm. Suppose that $M \in \mathbb{R}^{n\times n}$, $\lambda_{\min}(M)$ ($\lambda_{\max}(M)$) is the smallest (largest) real part of eigenvalues of the matrix M. Moreover, $\star$ is used to denote the symmetric part of a matrix, i.e., $\begin{bmatrix} a & b^T \\ b & c \end{bmatrix} = \begin{bmatrix} a & \star \\ b & c \end{bmatrix}$. The term $\text{Co}\{\cdot\}$ denotes the convex hull of a set.

II. Problem Setup

Consider the following linear uncertain systems:

$$\dot{x} = (A + \Delta A)x + (B + \Delta B)u + (B_w + \Delta B_w)w, \quad (1)$$

where $x(t) \in \mathbb{R}^{n_x}$ is the state of the system, $u(t) \in \mathbb{R}^{n_u}$ the control input. The signal $w(t) \in \mathbb{R}^{n_w}$ is the exogenous disturbance or uncertainty, which is unknown but bounded, and lies in a compact set,

$$\mathcal{W} := \{w \in \mathbb{R}^{n_w} \mid \|w\| \leq w_{max}\},$$

i.e., $w(t) \in \mathcal{W}$ for all $t \geq 0$. The system matrices $A \in \mathbb{R}^{n_x\times n_x}$, $B \in \mathbb{R}^{n_x\times n_u}$ and $B_w \in \mathbb{R}^{n_x\times n_w}$ are constant matrices, $\Delta A \in \mathbb{R}^{n_x\times n_x}$, $\Delta B \in \mathbb{R}^{n_x\times n_u}$ and $\Delta B_w \in \mathbb{R}^{n_x\times n_w}$ are compatible uncertain matrices. Equ. (1) is a set-valued function or a differential inclusion rather than a function.

Accordingly, the linear time invariant system:

$$\dot{x}(t) = Ax(t) + Bu(t) + B_w w(t), \quad (2)$$

is the "nominal" system of the given system (1).

Remark 2.1: Only 2-norm bounded disturbances are considered in this note. However, if $\mathcal{W} := \{w \in \mathbb{R}^{n_w} \mid \|w\|_\infty \leq \frac{1}{\sqrt{n_w}} \cdot w_{max}\}$ is chosen, the results proposed can be extended directly to deal with ∞-norm bounded disturbances.

Before proceeding, the definition of robust control invariant sets and a technique assumption on the system (2) are introduced. It shows that a robust (control) invariant set is a region of the state space such that the trajectory generated by the dynamical systems (under control) remains confined in the set if the initial state lies in it.

Definition 1: [3] A set $\Omega \subset \mathbb{R}^n$ is a robust control invariant set for the system (2) if there exists a feedback control law $\kappa(\cdot)$ such that for all $x(t_0) \in \Omega$ and for all $w \in \mathcal{W}$, $x(t) \in \Omega$ for all $t \geq t_0$.

Furthermore, if the control law $\kappa(\cdot)$ is determined, Ω is a *robust invariant set* of the closed-loop system.

Assumption 1: The pair (A, B) is stabilizable.

III. Robust Control Invariant Set based on Logarithmic Norm

In this section, the concept and characterization of logarithmic norm are introduced. Then, robust control invariant set based on logarithmic norm is discussed.

The logarithmic norm of a matrix M is defined as [27, 28]

$$\mu(M) = \lim_{h\to 0^+} \frac{\|I + hM\| - 1}{h}, \quad (3)$$

where the symbol $\|\cdot\|$ represents any matrix norm defined in the inner product space with inner product $\langle x, y\rangle$, and I is the compatible dimension identity matrix.

For the standard inner product $\langle x, y\rangle := x^T y$, the logarithmic norm of the matrix M is given by [29]

$$\mu_2(M) = \sup_{\|x\|=1} \text{Re}\langle x, Mx\rangle \quad (4)$$

where Re represents the real part of a complex number. Equ. (4) is equivalent to

$$\mu_2(M) = \lambda_{max}\left(\frac{M + M^T}{2}\right).$$

If the finitely many dimensions $\langle x, y\rangle := x^T H y$, where H is compatible positive definite matrix, then

$$\mu_H(M) = \max\left\{\lambda \mid \det(M^T H + HM - 2\lambda H) = 0\right\}, \quad (5)$$

where $\det(\cdot)$ is the determinant of given matrix [29]. Equ. (5) specifies $\mu_H(M)$ as a solution of a generalized eigenvalue problem. If $H = I$, it reduces to $\mu_2(M)$.

For the sake of computation purposes, Equ. (5) is reformulated as the form of matrix inequality:

$$\mu_H(M) = \min\left\{\beta \mid M^T H + HM - 2\beta H \leq 0\right\}, \quad (6)$$

Next lemma collects a subset of well-known results, the proof can be found in [28, 29], or reference therein.

Lemma 1: Let M and N be quadratic matrices, and $\lambda > 0$ denotes a positive real number. Then

- $\mu(\lambda M) = \lambda\mu(M)$,
- $\mu(M + N) \leq \mu(M) + \mu(N)$,
- $\|e^{Mt}\| \leq e^{\mu(M)t}$.

It concludes immediately from Lemma 1 that $\|e^{Mt}\| \leq 1$ for all nonnegative t if and only if $\mu(M) \leq 0$ [28, 29]. Equ. (4) and Equ. (5) show that logarithmic norm is *not* a real norm since it can be negative in some cases.

Since (A, B) is stabilizable, there exist a state feedback matrix $K \in \mathbb{R}^{n_u\times n_x}$ and positive matrix $P \in \mathbb{R}^{n_x\times n_x}$ such that $(A + BK)^T P + P(A + BK) \leq 0$ [30]. Compared with Equ. (6), in the case of $(A + BK)^T P + P(A + BK) \leq 0$, there exists $\mu_p(A + BK)$ such that $\mu_p(A + BK) \leq 0$, i.e., $\|e^{(A+BK)t}\|_p \leq 1$.

For simplicity, denote $A_{cl} := A + BK$. We can write the solution of the system (2) under the control law $u := Kx$ as

$$x(t) = e^{A_{cl}t}x(0) + \int_0^t e^{A_{cl}(t-\tau)} B_w w(\tau) d\tau \quad (7)$$

where $x(0)$ is the initial state of error system.

The exogenous disturbance $w(t)$ is bounded in the inner space $\langle x, y\rangle := x^T P x$ such that $\|w\|_p \le w_{p,\max}$ in terms of the equivalent induced matrix norm, where $w_{p,\max}$ is a given scalar. For example, if $\langle x, y\rangle = x^T H y$, then $w_{p,\max} \le \lambda_{\max}(P) w_{max}$.

The following theorem provides a way to construct robust control invariant sets of the system (2).

Theorem 1: Consider the system (2). Suppose that there exist matrices $K \in \mathbb{R}^{n_u \times n_x}$ and positive matrix $P \in \mathbb{R}^{n_x \times n_x}$ such that $\mu_p(A_{cl}) < 0$. Then, the set

$$\Omega := \left\{ x \in \mathbb{R}^{n_x} \mid \|x\|_p \le \frac{\|B_w\|_p w_{p,\max}}{-\mu_p(A_{cl})} \right\} \tag{8}$$

is a robust invariant set of the system (2).

Proof: The inequality $\|e^{A_{cl} t}\|_p \le e^{\mu_p(A_{cl} t)}$ is used to estimate the solution (7)*:*

$$\begin{aligned} \|x(t)\|_p &\le \left\|e^{A_{cl} t} x(0)\right\|_p + \left\| e^{A_{cl}(t-\tau)} B_w w(\tau) d\tau \right\|_p, \\ &\le e^{\mu_p(A_{cl} t)} \|x(0)\|_p + \left(1 - e^{\mu_p(A_{cl} t)}\right) \frac{\|B_w\|_p w_{p,\max}}{-\mu_p(A_{cl})}, \\ &= e^{\mu_p(A_{cl})} \left(\|x(0)\|_p - \frac{\|B_w\|_p w_{p,\max}}{-\mu_p(A_{cl})} \right) + \frac{\|B_w\|_p w_{p,\max}}{-\mu_p(A_{cl})}. \end{aligned}$$

Thus, if $x(0) \in \Omega$, then $x(t) \in \Omega$ for all $t \ge 0$. Therefore, Ω is an invariant set of the system (2). □

Remark 3.1: If there exists a matrix $K \in \mathbb{R}^{n_u} \times \mathbb{R}^{n_x}$ such that $\mu_2(A + BK) \le 0$ for the pair (A, B), then, the closed-loop system has the “optimal” transient behavior since $\left\|e^{At}\right\| \le M e^{\mu_2(A+BK)}$ and $M = 1$.

The forgoing discussion also gives a valuable insight into part b) of Problem 1, in the chapter of “Problem 6.3” [31], for the system which has the property of $\mu_2(A + BK) \le 0$.

Remark 3.2: The linear system (2) is exponentially stable if $w(t) \equiv 0$ and $\mu_p(A_{cl}) < 0$, since $A_{cl}^T P + P A_{cl} \le 2\mu_p(A_{cl}) P$. Theorem 1 shows that there exists a control invariant set if the system (2) is exponentially stable.

Remark 3.3: If standard inner product $\langle x, y\rangle = x^T y$ is adopted, there is *no* guarantee that there exists a matrix $K \in \mathbb{R}^{n_u} \times \mathbb{R}^{n_x}$ such that $\mu_2(A + BK) \le 0$ for any pair (A, B) even if (A, B) is stabilizable.

Theorem 1, together with Remark 3.2, show that there exists a control invariant set if the corresponding nominal systems are exponentially stable. In the next section, robust control invariant sets of linear systems with uncertainty are considered.

IV. Robust Control Invariant Set based on Differential Inequality

Definition 2: [30] A continuous function $\alpha : [0, a) \to [0, \infty)$ is said to belong to class $\mathcal{K}$ if it is strictly increasing and $\alpha(0) = 0$. It is said to belong to class $\mathcal{K}_\infty$ if $a = \infty$ and $\alpha(r) \to \infty$ as $r \to \infty$.

Lemma 2: [16] Let $S : \mathbb{R}^{n_x} \to [0, \infty)$ be a continuously differentiable function and $\alpha_1(\|v\|) \le S(v) \le \alpha_2(\|v\|)$. Suppose there exist $\lambda > 0$ and $\mu > 0$ such that

$$\frac{d}{dt} S(v) + \lambda S(v) - \mu w^T w \le 0, \quad \forall w \in \mathcal{W}, \tag{10}$$

where α_1, α_2 are class $\mathcal{K}_\infty$ functions. Then, the system trajectory starting from $v(t_0) \in \Omega$ will remain in the set Ω, where

$$\Omega := \left\{ v \in \mathbb{R}^{n_x} \mid S(v) \le \frac{\mu w_{max}^2}{\lambda} \right\}. \tag{11}$$

The following proof gives an interpretation of Lemma 2 from the optimization theory perspective.

Proof: For the system (2)*, $x \notin \Omega$ is equivalent to $S(x) > \frac{\mu w_{\max}}{\lambda}$, and $w \in \mathcal{W}$ is equivalent to $w(t)^T w(t) \le w_{\max}^2$ for all $t \ge 0$. In terms of the S-procedure [32], it is sufficient for $\dot{S}(x) \le 0$ for all $x \notin \Omega$ and for all $w \in \mathcal{W}$, if it holds that*

$$-\frac{d}{dt} S(x) - \lambda \left(S(x) - \frac{\mu w_{\max}^2}{\lambda} \right) - \mu \left(w_{\max}^2 - w(t)^T w(t) \right) \ge 0,$$

with $\mu > 0$ and $\lambda > 0$. That is, $\frac{d}{dt} S(x) + \lambda S(x) - \mu w^T w \le 0$, for all $w(t) \in \mathcal{W}$. □

V. An Extension to Linear Uncertain Systems

Our goal is to find a robust control invariant set (1) for the given differential inclusion.

A. Polytopic Model of Linear Uncertain Systems

If $[A + \Delta A, B + \Delta B, B_w + \Delta B_w] \in \Sigma$, where

$$\Sigma := \text{Co}\left\{ \begin{bmatrix} A_1 & B_1 & B_{w,1} \end{bmatrix}, \ldots, \begin{bmatrix} A_N & B_N & B_{w,N} \end{bmatrix} \right\}, \tag{12}$$

then, Σ is a polytopic model of the system (1), $\begin{bmatrix} A_i & B_i & B_{w,i} \end{bmatrix}$, $i \in \mathbb{Z}_{[1,N]}$, is the vertex matrix of the set Σ, and N is the number of vertex matrix.

Theorem 2: Suppose that there exist a positive definite matrix $X \in \mathbb{R}^{n_x \times n_x}$, a non-square matrix $Y \in^{n_u \times n_x}$, and scalars $\lambda > 0$ and $\mu > 0$ such that

$$\begin{bmatrix} (A_i X + B_i Y)^T + A_i X + B_i Y + \lambda X & B_{w,i} \\ * & -\mu I \end{bmatrix} \le 0, \tag{13}$$

for all $i \in \mathbb{Z}_{[1,N]}$. Then, with $u(t) := Kx(t)$ and $S(x(t)) := x(t)^T P x(t)$, where $P := X^{-1}$ and $K := Y X^{-1}$, inequality (10) is satisfied for the PLDI (12). Therefore, the system (1) is robustly invariant in the set

$$\Omega := \left\{ x \in \mathbb{R}^{n_x} \mid x^T P x \le \frac{\mu w_{max}^2}{\lambda} \right\}.$$

Proof: Pre-and post-multiplying (13) by $\text{diag}(P, I)$ yields

$$\begin{bmatrix} (A_i + B_i K)^T P + P(A_i + B_i K) + \lambda P & P B_{w,i} \\ * & -\mu I \end{bmatrix} \le 0, \tag{14}$$

for all $i \in \mathbb{Z}_{[1,N]}$. Multiplying (14) *from both sides with $\begin{bmatrix} v(t) & w(t) \end{bmatrix}$ and $\begin{bmatrix} v^T(t) & w^T(t) \end{bmatrix}^T$, respectively, due to* (1)*, it follows that the inequality $\frac{d}{dt}(v(t)^T P v(t)) + \lambda v(t)^T P v(t) - \mu w(t)^T w(t) \le 0$ is satisfied for all $w(t) \in \mathcal{W}$. Therefore, inequality (10) holds for the system (1).* □

B. Norm-bounded Model of Linear Uncertain Systems

If $[A+\Delta A, B+\Delta B, B_w+\Delta B_w] \in \Sigma$, where

$$\Sigma := \{(\mathcal{A},\mathcal{B},B_w) \mid \mathcal{A} = A + M\Delta(t)N_1, \\ \mathcal{B} = B + M\Delta(t)N_2, \mathcal{B}_w = B_w + M\Delta(t)N_3B_w\},$$

M, N_1 and N_2 are known matrices of appropriate dimensions and $\Delta(t)$ is a time-varying norm-bounded matrix satisfying

$$\bar{\sigma}(\Delta(t)) \leq 1,$$

then, Σ is a norm-bounded model of the system (1).

In the proof of the next lemma, the following lemma is needed.

Lemma 3: [33, 34] Given matrices Y, H, E of appropriate dimensions and with Y symmetrical, then

$$Y + HFE + E^TF^TH^T \leq 0$$

for all F satisfying $F^TF \leq I$, if and only if there exists a scalar $\epsilon > 0$ such that

$$Y + \epsilon HH^T + \epsilon^{-1}E^TE \leq 0.$$

Parallel to Lemma 2, we have the following lemma:

Theorem 3: Suppose that there exist a positive definite matrix $X \in \mathbb{R}^{n_x \times n_x}$, a non-square matrix $Y \in^{n_u \times n_x}$, and scalars $\lambda > 0$, $\mu > 0$ and $\epsilon > 0$ such that

$$\begin{bmatrix} \Gamma & B_w & (N_1X+N_2Y) \\ * & -\mu I & N_3 \\ * & * & -\epsilon I \end{bmatrix} \leq 0. \quad (15)$$

Then, with $u := Kx$ and $S(x) := x^TPx$, where $\Gamma := (AX+BY)^T + AX + BY + \lambda X + \epsilon MM^T$, $P := X^{-1}$ and $K := YX^{-1}$, inequality (10) is satisfied for the system (1). Therefore, the system (1) is robustly invariant in the set $\Omega := \left\{x \in \mathbb{R}^{n_x} \mid x^TPx \leq \frac{\mu w_{max}^2}{\lambda}\right\}$.

Proof: Congruence (15) *with* diag$\{P, I, I\}$*, we have*

$$\begin{bmatrix} P\Gamma P & PB_w & (N_1+N_2K) \\ * & -\mu I & N_3 \\ * & * & -\epsilon I \end{bmatrix} \leq 0. \quad (16)$$

By the Schur complement, (16) *is equivalent to*

$$\begin{bmatrix} (A+BK)^TP + P(A+BK) + \lambda P & PB_w \\ * & -\mu I \end{bmatrix} \\ + \epsilon^{-1}\begin{bmatrix} N_1+N_2K \\ N_3 \end{bmatrix}\begin{bmatrix} (N_1+N_2K)^T & N_3 \end{bmatrix} \\ + \epsilon \begin{bmatrix} PM \\ 0 \end{bmatrix}\begin{bmatrix} M^TP & 0 \end{bmatrix} \leq 0.$$

Since $\bar{\sigma}(\triangle(t)) \leq 1$ *and* $\epsilon > 0$*, due to Lemma 3, the forgoing equation is equivalent to*

$$\begin{bmatrix} \prod & PB_w \\ * & -\mu I \end{bmatrix} \leq 0,$$

where $\prod := (A + M\triangle N_1 + (B + M\triangle N_2)K)^T P + P(A + M\triangle N_1 + (B + M\triangle N_2)K) + \lambda P$*. Multiplying* (15) *from both sides with* $\begin{bmatrix} v(t) & w(t) \end{bmatrix}$ *and* $\begin{bmatrix} v^T(t) & w^T(t) \end{bmatrix}^T$*, respectively, inequality (10) holds for the system (1).* □

C. Optimization of the robust control invariant set

In order to reduce the effect of disturbances or perturbations, the minimal robust control invariant set is recommended. The volume of ellipsoid centered at the origin Ω is proportional to $\det\left(\frac{\mu w_{\max}^2}{\lambda}X\right)$ [32]. The geometric mean of the eigenvalues [35] which leads to minimization of $\det(\alpha X)^{\frac{1}{n_x}}$, can be used for solving the determinant maximization problem.

The minimization problem of the ellipsoid Ω can be formulated as

$$\begin{aligned} &\underset{X,Y,\lambda,\mu}{\text{maximize}} \ \det\left(\frac{\mu w_{\max}^2 X}{\lambda}\right)^{\frac{1}{n_x}} \\ &\text{subject to} \quad (13) \end{aligned} \quad (17)$$

or

$$\begin{aligned} &\underset{X,Y,\lambda,\mu,\epsilon}{\text{maximize}} \ \det\left(\frac{\mu w_{\max}^2 X}{\lambda}\right)^{\frac{1}{n_x}} \\ &\text{subject to} \quad (15) \end{aligned} \quad (18)$$

Remark 5.1: The optimization problems (17) and (18) are not linear matrix inequalities (LMIs) optimization problems since there exists terms of λX and $\frac{\mu w_{\max}^2 X}{\lambda}$. In order to find a possible minimum robust control invariant set by LMI toolbox, a search over λ and μ, or λ, μ and ϵ, is required, respectively.

VI. Conclusions

In this paper, schemes for the computation of robust control invariant sets are proposed for systems with norm-bounded uncertainties or polytopic uncertainties and additive disturbances. Logarithmic norm as well as the functional inequality is used to design the robust control invariant set along with their corresponding linear state feedback control law. It shows that there exists a robust control invariant set if the system without additive disturbances is exponentially stable, although it is not necessarily a minimal invariant set. Furthermore, the robust control invariant set and the corresponding control law can be solved through a single LMI optimization problem.

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References

[1] I. Kolmanovsky and E. Gilbert, "Theory and computation of disturbance invariant sets of discrete-time linear systems," *Math. Probl. Eng.*, vol. 4, no. 4, pp. 317–367, 1998.

[2] F. Tahir and I. M. Jaimoukha, "Roubst positively invariant sets for linear systems subject to model-uncertainty and disturbances," in *4th IFAC Nonlinear Model Predictive Control Conference*, Noordwijkerhout, NL., August 23-27, 2012, pp. 69–73.

[3] F. Blanchini, "Set invariance in control," *Automatica*, vol. 35, no. 11, pp. 1747–1767, 1999.

[4] E. C. Kerrigan, "Robust Constraint Satisfaction: Invariant Sets and Predictive Control," Ph.D. Thesis, University of Cambridge, 2000.

[5] E. C. Kerrigan and J. M. Maciejowshki, “Invariant sets for constrained nonlinear discrete-time systems with application to feasibility in model predictive control,” in *Proc. Amer. Contr. Conf.* IEEE, 2000, pp. 4951–4956.
[6] F. Blanchini and S. Miani, *Set-Theoretic Methods in Control*. Boston: Springer, 2007.
[7] M. V. Kothare, V. Balakrishnan, and M. Morari, “Robust constrained model predictive control using linear matrix inequalities,” *Automatica*, vol. 32, no. 10, pp. 1361–1379, 1996.
[8] H. Chen and C. W. Scherer, “Disturbance attenuation with actuator constraints by moving horizon H_∞ control,” in *IFAC International Symposium on Advanced Control of Chemical Processes*, HongKong, China, 2003, pp. 447–452.
[9] H. Chen and K.-H. Guo, “Constrained H_∞ control of active suspensions: An LMI approach,” *IEEE Contr. Syst. Tech.*, vol. 13, no. 3, pp. 412–421, 2005.
[10] D. Q. Mayne, J. B. Rawlings, C. V. Rao, and P. O. M. Scokaert, “Constrained model predictive control: stability and optimality,” *Automatica*, vol. 36, no. 6, pp. 789–814, 2000.
[11] H. Chen and F. Allgöwer, “A quasi-infinite horizon nonlinear model predictive control scheme with guaranteed stability,” *Automatica*, vol. 34, no. 10, pp. 1205–1217, 1998.
[12] W. H. Chen, J. O’Reilly, and D. J. Ballance, “On the terminal region of model predictive control for nonlinear systems with input/state constraints,” *Int. J. Adaptive Control and Signal Process*, vol. 17, no. 3, pp. 195–207, 2003.
[13] S.-Y. Yu, H. Chen, P. Zhang, and X.-J. Li, “An LMI optimization approach for enlarging the terminal region of NMPC,” *Acta Automatica Sinca*, vol. 34, no. 7, pp. 798–804, 2008.
[14] S.-Y. Yu, H. Chen, and X.-J. Li, “Enlarging the terminal region of NMPC based on T-S fuzzy model,” *International Journal of Control, Automation and Systems*, vol. 7, no. 3, pp. 481–486, 2009.
[15] D. Q. Mayne, M. M. Seron, and S. V. Rakovic, “Robust model predictive control of constrained linear systems with bounded disturbances,” *Automatica*, vol. 41, no. 2, pp. 219–224, 2005.
[16] S.-Y. Yu, C. Böhm, H. Chen, and F. Allgöwer, “Robust model predictive control with disturbance invariant sets,” in *Proc. Amer. Contr. Conf.*, Baltimore, Maryland, 2010, pp. 6262–6267.
[17] S.-Y. Yu, C. Maier, H. Chen, and F. Allgöwer, “Tube MPC scheme based on robust control invariant set with application to lipschitz nonlinear systems,” vol. 62, no. 2, pp. 194–200, 2013.
[18] S. V. Rakovic, B. Kouvaritakis, M. Cannon, and C. Panos, “Fully parameterized tube model predictive control,” *Int. J. of Robust and Nonlinear Control*, vol. 22, no. 12, pp. 1330–1361, 2012.
[19] S. V. Rakovic, B. Kouvaritakis, M. Cannon, C. Panos, and R. Findeisen, “Parameterized tube model predictive control,” *IEEE Trans. Automat. Contr.*, vol. 57, no. 11, pp. 2746–2761, 2012.
[20] K. M. Zhou, *Essentials of Robust Control*. Upper Saddle River, New Jersey: Prentice Hall, 1999.
[21] E. Kofman, H. Haimovich, and M. M. Sermon, “A systematic method to obtain ultimate bounds for perturbed systems,” *Int. J. Contr.*, vol. 80, no. 2, pp. 167–178, 2007.
[22] S. V. Rakovic, E. C. Kerrigan, K. I. Kouramas, and D. Q. Mayne, “Invariant approxiamtion of the minimal robust positively invariant set,” *IEEE Trans. Automat. Contr.*, vol. 50, no. 3, pp. 406–410, 2005.
[23] S. V. Rakovic, E. C. Kerrigan, D. Q. Mayne, and K. I. Kouramas, “Optimized robust control invariance for linear discret-time systems: Theoretical foundations,” *Automatica*, vol. 43, no. 5, pp. 831–841, 2007.
[24] S. V. Rakovic, A. R. Teel, D. Q. Mayne, and A. Astolfi, “Simple robust control invariant tubes for some classes of nonlinear discrete time systems,” in *Proc. 45th IEEE Conf. Decision Contr.*, San Diego, USA, 2006, pp. 6397–6402.
[25] M. Korda, D. Henrion, and C. N. Jones, “Convex computation of the maximum controlled invariant set for discret-time polynomial control systems,” in *Proc. 52th IEEE Conf. Decision Contr.*, Florenc, Italy, 2013, pp. 7107–7112.
[26] M. Fiacchini, T. Alamo, and E. F. Camacho, “On the computation of convex robust control invariant sets for nonlinear systems,” *Automatica*, vol. 46, no. 8, pp. 1334–1338, 2010.
[27] G. Dahlquist, *Stability and Error Bounds in the Numerical Integration of Ordinary Differential Equations*. Kungl. Tekniska Högskolaus Handlingar, Stockholm, Nr. 130, 1959.
[28] K. Dekker and J. G. Verwer, *Stability of Runge-Kutta Methods for Stiff Nonlinear Differential Equations*. Amsterdam: North-Holland, 1984.
[29] T. Ström, “On logarithmic norms,” *SIAM J. Numer. Anal.*, vol. 12, no. 5, pp. 741–753, 1975.
[30] H. K. Khalil, *Nonlinear Systems (third edition)*. New York: Prentice Hall, 2002.
[31] V. Blondel and A. Megretski, *Unsolved Problems in Mathematical Systems and Control Theory*. Princeton, New Jersey: Princeton University Press, 2004.
[32] S. Boyd, L. El Ghaoui, E. Feron, and V. Balakishnan, *Linear Matrix Inequalities in System and Control Theory*. Philadelphia: SIAM, 1994.
[33] P. P. Khargonekar, I. R. Petersen, and K. Zhou, “Robust stabilization of uncertain linear systems: quadratic stabilizability and H_∞ control theory,” *IEEE Trans. Automat. Contr.*, vol. 35, no. 3, pp. 356–361, 1990.
[34] L. H. Xie, “Output feedback H_∞ control of systems with parameter uncertainty,” *Int. J. Contr.*, vol. 63, no. 4, pp. 741–750, 1996.
[35] Y. Nesterov and A. Nemirovsky, *Interior point polynomial methods in convex programming*. Philadelphia, USA: SIAM Publications, 1994.